Statistical analysis of the supersymmetry breaking scale

Michael R. Douglas[1,&,2]

[1] NHETC and Department of Physics and Astronomy
Rutgers University,
Piscataway, NJ 08855-0849, USA
[&] I.H.E.S., Le Bois-Marie, Bures-sur-Yvette, 91440 France
[3] Caltech 452-48, Pasadena, CA 91125, USA

`mrd@physics.rutgers.edu`

Abstract

We discuss the question of what type and scale of supersymmetry breaking might be statistically favored among vacua of string/M theory, building on comments in Denef and Douglas, hep-th/0404116.

1 Introduction

Over the last few years, a point of view towards the phenomenology of string and M theory has evolved [9, 6, 14, 3, 10, 11, 17, 20, 12, 2, 4, 7, 16, 8, 21], which develops and tries to work with the following hypotheses:

- String/M theory has many consistent vacuum states which at least roughly match the Standard Model, and might be candidates to describe our world.
- The number of vacua is so large that the problems of reproducing the Standard Model in detail, and the classic problems of "beyond the Standard Model physics" such as the hierarchy problem and cosmological constant problem, might admit statistical solutions. The basic example is that in an ensemble of N vacua which differ only in a parameter Λ (say the c.c. as in [6]), and in which Λ is uniformly distributed, it is likely that a quantity which appears fine tuned by an amount $\epsilon > 1/N$ (for the c.c., 10^{-120} in a generic nonsupersymmetric theory) will be realized by at least one vacuum, just on statistical grounds.
- No single vacuum is favored by the theory. Although selection principles might be found, they will not determine a unique vacuum *a priori*, but rather cut down the possibilities in a way which is useful only when combined with other information.

These three hypotheses are very general and could in principle apply to any candidate "theory of everything." As such, they were studied in pre-string work in quantum cosmology. The new ingredient is the fourth hypothesis:

- String/M theory contains a precisely (and someday even mathematically) defined set of vacua, and this set has a great deal of structure which we can to some extent understand.

The new idea is that we need to understand this set of possible vacua, and the larger configuration space containing it, to make meaningful predictions and test the theory. Various approaches to making predictions are under study by various authors, some combining stringy and anthropic considerations, others purely statistical. In any case, one de-emphasizes the study of individual vacua, in favor of studying distributions of large classes of vacua.

This structure containing the vacua is often called the stringy "landscape" [20], a term which has long been used in molecular physics, quantum cosmology, and even mathematical biology (fitness landscapes), for describing potentials or other functions to be optimized in high dimensional configuration spaces. It is a catchy and useful term, so long as one keeps in mind that the study of the effective potential is only one aspect of the problem.

Although all of these hypotheses have roots in the early study of string theory and in quantum cosmology, and in various forms this point of view has been entertained by many physicists, it seems fair to say that string theorists have long hoped (indeed most still do) that this would **not** be the case, that either

one of the remaining problems in constructing a realistic vacuum would prove so constraining that it alone would rule almost all vacua out of consideration, or that some fundamental new insight would radically change the problem and make all considerations based on our present pictures of string compactification irrelevant. The main "conceptual leap" required to adopt the new point of view is simply to abandon these (so far not very productive) beliefs and instead try to work with the actual results which have come out of string compactification, which to our mind are quite consistent with the hypotheses above.

It might even turn out that progress along these lines would reconcile the old and new points of view. After all, the problem of finding a fully satisfactory vacuum is highly constraining; combining the idea that c.c. must be tuned, with the many other tests that must be passed, strongly suggests that the fraction of vacua which work is significantly less than 10^{-60}. We do not know yet that there are even 10^{60} consistent vacua. What does follow from the present works is that this "acceptable" number of vacua would only emerge by first making a rough analysis, not enforcing all tests and consistency conditions, which leads to a embarrassingly large number of proto-vacua, and then placing many cuts and consistency conditions on these proto-vacua.

Considering this problem is another way to motivate a statistical approach, in which one can benefit from the hypothesis than many of the tests and consistency conditions are to a good approximation independent, meaning that the fraction of models which pass two tests is the product of the fractions which pass each test. While clearly this is not always true, it is very plausible for many pairs of tests. For example, while each specific vacuum has a definite number of generations of quarks and leptons, and a definite cosmological constant, both of which must agree with observation, there is no reason to think that these two numbers have any correlation in the distribution of all vacua. Extrapolating this type of logic as in [12], one starts to feel it should be far easier to get useful estimates of the number of compactifications which work, than to explicitly construct even one complete example.

After setting out some of these ideas in [12, 2], I talked to many phenomenologists to ask what actual statistics of vacua they might be interested in finding out about.[1] The most common answer was to learn about the likelihood of low energy supersymmetry and more specifically the scale of supersymmetry breaking.

2 Observations on supersymmetry breaking scales

Some general considerations were stated in [12], and more specific arguments favoring low scale supersymmetry were given in [4].

In [7], Denef and I studied supersymmetric and non-supersymmetric flux vacua on CY's with one complex structure modulus in detail, and based on these results made the following comments in our conclusions about this problem.

[1] Particular thanks go to S. Dimopoulos, G. Kane, S. Thomas and M. Wise.

Our considerations were based on the assumption that supersymmetry breaking is spontaneous and described by the usual $N = 1$ supergravity Lagrangian, in particular the potential

$$V = e^{\mathcal{K}/M_p^2} \left(g^{i\bar{j}} D_i W D_{\bar{j}} W^* - \frac{3}{M_p^2} |W|^2 \right) + \sum_\alpha D_\alpha^2. \qquad (1)$$

Most of our detailed considerations were for flux vacua in IIb string theory. [15] However, there are simple intuitive explanations for the claims, so they could well be more general.

Let us consider a joint distribution of vacua

$$d\mu[F_i, D_\alpha, \hat{\Lambda}] = \prod dF\ dD\ d\hat{\Lambda} \rho(F_i, D_\alpha, \hat{\Lambda}).$$

where $\hat{\Lambda} = 3e^K |W|^2$ is the norm of the superpotential, and $F_i = D_i W$ are the F breaking parameters (auxiliary fields). The generic vacuum with unbroken supersymmetry would be AdS with cosmological constant $\Lambda = -\hat{\Lambda}$. More generally, the c.c. would be

$$\Lambda = \sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - \hat{\Lambda} \qquad (2)$$

and thus the constraint that a physical vacuum should have $\Lambda \sim 0$ can be expressed as a delta function. Thus, we can derive a distribution of supersymmetry breaking scales under the assumption of near zero cosmological constant, as

$$d\mu_{\Lambda=0}[F_i, D_\alpha] = \prod d^2 F_i\ dD_\alpha\ d\hat{\Lambda} \rho(F_i, D_\alpha, \hat{\Lambda})\ \delta(\sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - \hat{\Lambda}). \quad (3)$$

Of course, we could solve the delta function for one of the variables.

Now, a main observation of [7] (section 3.3; this was suggested earlier by more heuristic arguments [13, 18]) is that the distribution of values of $\hat{\Lambda}$ is uniform near zero, and throughout its range is more or less uniform.[2] Furthermore it is relatively uncorrelated with the supersymmetry breaking parameters.

The simple physical argument for this is that the superpotential W (and perhaps K as well) gets additive contributions from many sectors of the theory, supersymmetric and nonsupersymmetric. If there are many supersymmetric hidden sectors, varying choices in the hidden sectors (say of flux) will vary $\hat{\Lambda}$ in a way which is uncorrelated with the supersymmetry breaking, and will tend to produce structureless, uniform distributions.

Thus, we can proceed by solving for $\hat{\Lambda}$ in Eq. (3). Since its distribution is uniform and uncorrelated with F and D, the distribution function ρ is in fact independent of $\hat{\Lambda}$, and this simply leads to

$$d\mu_{\Lambda=0}[F_i, D_\alpha] = \prod d^2 F_i\ dD_\alpha\ \rho(F_i, D_\alpha). \qquad (4)$$

[2] v2: The reason it is $|W|^2$, and not $|W|$ or some other power which is uniformly distributed, is because (in [7] and perhaps more generally) W is uniformly distributed as a complex variable. Then, writing $W = e^{i\theta}|W|$, we have $d^2W = \frac{1}{2} d\theta d(|W|^2)$.

In other words, the constraint of setting the c.c. to a near zero value, has no effect on the resulting distribution, because we had a wide variety of supersymmetric effects available to compensate the supersymmetry breaking contributions.

In itself, this does not prefer a particular scale of supersymmetry breaking, rather it says that the condition of tuning the c.c. does **not** lead to a factor of Λ/M_{susy}^4 favoring low supersymmetry breaking scales; instead it is neutral with regard to supersymmetry breaking scale.

We also pointed out that, given many supersymmetry breaking parameters, the high scale becomes favored. This simply follows from the idea that[3]

$$\begin{aligned} d\mu[M_{susy}^2] \quad &= \int \prod_{i=1}^{n_F} d^2F_i \ \prod_{\alpha=1}^{n_D} dD_\alpha \ d(M_{susy}^4)\delta(M_{susy}^4 - \sum_i |F_i|^2 - \sum_\alpha D_\alpha^2) \\ &\sim (M_{susy}^2)^{2n_F+n_D-1} d(M_{susy}^2) \end{aligned} \tag{5}$$

In words, the total supersymmetry breaking scale is the distance from the origin in the space of supersymmetry breaking parameters, and in a high dimensional space most of the volume is near the boundary. Essentially the same observation is made independently by Susskind in [21].

This argument is particularly simple if we assume the distributions of the individual breaking parameters are uniform. This is more or less what came out in the one modulus F breaking distributions we studied in [7], but we only considered special limits and this is probably simplistic. More generally, it is quite likely that more complicated regimes or multi-modulus models will contain vacua with supersymmetry breaking at hierarchically small scales.

First, the number of vacua in which small scales are generated is large, although not overwhelmingly so. In flux compactifications, this is because of the enhancement of the number of vacua near conifold points discussed in section 3.1.4 of [7] (see also [16]). While in the one parameter models discussed in [7], these always come with tachyons (in D breaking models), this phenomenon stemmed from the specific structure of the mass matrix in one parameter models and is not expected to hold in general.

Second, these effects can drive supersymmetry breaking both by F and D terms, as seen in many concrete models, for example by putting antibranes in the conifold throat as discussed in [17, 21]. It seems reasonable to suppose that vacua with this type of supersymmetry breaking are common, although this claim remains to be checked.

This should skew the distribution towards small scales but not overwhelmingly so, as the vast majority of vacua (as seen explicitly in [7]) do not produce exponentially small scales. This is the sense in which the uniform distribution is still a reasonable picture.

In any case, the underlying observation that adding together many positive breaking terms as in Eq. (5) will produce a distribution weighed towards high scales is rather general.

[3] v2: We added D breaking parameters as well; v3: fixed typo.

3 Further considerations

Recently Arkani-Hamed and Dimopoulos have outlined a fairly detailed phenomenological scenario using high scale supersymmetry breaking [1]. Rather than the boring prediction that we will only see the Higgs at LHC, they argue for a very interesting alternate scenario in which gauginos and Higgsinos could be seen at LHC, mostly on the grounds that keeping these particles light could duplicate the famous agreement of supersymmetric grand unification with precision gauge coupling measurements (see [5] for a recent discussion) in a novel, superficially non-supersymmetric scenario.

While a testable phenomenological scenario is its own justification, it is also interesting to ask if string theory might favor or disfavor the new scenario, or better, whether we have any hope of making any justifiable statement about this question in the near future from string theory at all.

Now the considerations we just outlined speak against one of the motivating arguments suggested in [1]. Namely, the idea that tuning the c.c. is the dominant consideration in analyzing the number of models. In the models we discussed, and for the general reasons we discussed, tuning the c.c. is more or less independent of other considerations.

On the one hand, we have the standard advantage of low scale supersymmetry in solving the hierarchy problem. One would naively guess and in [1] arguments are given that the fraction of vacua which work (or tuning factor) is

$$\rho(M_H, M_{susy}) \propto M_H^2/M_{susy}^2. \tag{6}$$

I believe this is correct.[4]

On the other hand, we could have many more high scale models, despite the neutrality of the c.c. considerations, just because of the many possible supersymmetry breaking parameters, as expressed in Eq. (5).

What would we need to do to make this precise? First, we need to distinguish the question of the overall supersymmetry breaking scale, from that of the observable supersymmetry breaking scale, which enters into the solution of the hierarchy problem.

The overall supersymmetry breaking scale, which is essentially $\hat{\Lambda}$ defined above, determines the gravitino mass. It is the scale which is determined to be high by the previous arguments. Again, there is a simple intuitive explanation for this claim. It is that string compactifications typically contain many hidden sectors, and there is nothing in our considerations that disfavors supersymmetry breaking in hidden sectors at a high scale. Thus one is led to the general prediction that the gravitino is typically very heavy, more or less independently of observed physics.

The question of how the many supersymmetry breaking parameters enter into the observable sector clearly needs a lot of work to understand. Many mechanisms have been proposed in the literature, and we are now asking for

[4] v2: L. Susskind has suggested that even this should not be taken for granted, because of the well known μ problem.

some distribution over models which realize the various mechanisms. This probably requires knowing something about the distributions of gauge groups and matter content, along the lines discussed in [12].

Since all the basic scenarios discussed in textbooks [19] are driven by the expectation value of a single auxiliary field, as a first guess, one is tempted to say that the parameter which controls supersymmetry breaking in the observable sector, and thus enters into Eq. (6), will be a single F or D term, in other words the expectation value of the auxiliary component of a single field.

As discussed in the previous section, a reasonable approximation for the distribution of this term is that it is uniform. In this case, vacuum statistics would definitely favor low scale supersymmetry, more or less by the standard argument. Indeed, as the standard lore would have it and as we discussed, the real distribution appears to be somewhat skewed towards producing exponentially small hierarchies, which further favors the low scale scenarios.

On the other hand, it could be that in the typical embeddings of the MSSM and its cousins in string theory, the Higgs mass is determined by the sum of several positive parameters, analogous to the sum of squares appearing in Eq. (5), but perhaps fewer in number. The resulting distribution would be Eq. (5) multiplied by Eq. (6). In this case, the high scale would be favored, even with two F parameters, which would lead to $\rho \sim M_{susy}^4 d(M_{susy}^2)$.

Taking into account the effects favoring hierarchies and all the other complexities of the problem clearly requires a more careful treatment. An important point which we probably do not understand well enough to do this properly, is what places the cutoff on the maximal realized scales of supersymmetry breaking. One might guess the Planck scale or the string scale, but the results will probably be sensitive to this guess. The specific cutoff which enters the flux vacuum counting results in [7] is the string scale multiplied by a "O3 charge" which is quantized and can reach $L \sim 1000$ in examples; this is a maximum bound which neglects many other possible effects.

While this may well be a good estimate for the cutoff on the W distribution (since this receives contributions from supersymmetric sectors), it seems likely that the cutoff on supersymmetry breaking scales could be lower. This is suggested by the very strong intuition among string theorists that stable non-supersymmetric models are difficult to construct, which suggests a cutoff determined by considering stability.

A clear example of this can be seen by considering the masses of moduli in D-breaking scenarios, as discussed in [8] section 4. The point made there is the (surely well known) observation that the bosonic mass matrix in a pure D breaking vacuum is

$$M^2 = H(H - 3e^{K/2}|W|)$$

where $H = dd(e^{K/2}|W|)$ is the fermion mass matrix. This is the generalization of the usual bose-fermi mass relation to a supersymmetric AdS vacuum, which by assumption is the situation before D breaking, and the D breaking terms are expected not to change this much. Thus one finds that tachyons correspond to fermion masses between 0 and $3e^{K/2}|W| = (3\hat{\Lambda})^{1/2}$, and the higher the scale

of supersymmetry breaking, the more likely are tachyons. In itself this effect probably only lowers the peak of the distribution by a factor $1/n$ where $n \sim 100$ is the number of moduli, but it is conceivable that other instabilities are important and drive the scale lower still.

A reasonable summary of the suggestions we have come to is that in models of supersymmetry breaking which are driven by a single parameter (F or D term), one should expect low scale breaking, while in models in which the scale of supersymmetry breaking entering into Eq. (6) is the sum of squares (or similar combinations) of more than one independently distributed parameter (more precisely, $2n_F + n_D \geq 3$), one could expect high scale breaking.[5]

4 Conclusions

It seems evident that the new approaches to stringy phenomenology outlined in the introduction, which in the form we are pursuing could be called the "statistical approach," are stimulating interesting developments and broadening the range of assumptions used in model building.

I tried to outline a few of the issues which would go into actually finding the distribution of supersymmetry breaking scales. Many more are discussed in the references.

At this point, it is not at all obvious whether high or low scales will be preferred in the end. We explained how this depends on the coupling of supersymmetry breaking to the observable sector, and using a simple ansatz of uniform distributions we stated a model building criterion for cases which could favor the high scale – it would be expected if the definition of "supersymmetry breaking scale" which controls the Higgs mass (this need not be the gravitino mass) depends on the sum of independently distributed positive breaking parameters (one needs the numbers of F and D parameters to satisfy $2n_F + n_D \geq 3$).

We should keep in mind that "favoring" one type of vacuum or mechanism over another is not a strong result, if both types of vacuum exist. It might be that other considerations such as cosmology pick the less numerous type. It might be that we just happen to live in one of the less numerous types. On the other hand, if the actual numbers of vacua are not too large, falsifiable claims could come out of string theory using this approach. This remains to be seen.

Note added in v3. In [22], Susskind has reviewed these arguments and suggested that they already predict that supersymmetry will not be seen at the TeV scale. This conclusion seems premature as changing some of the assumptions will reverse the conclusion. One is the number of breaking parameters discussed above. Another is that if the breaking parameters add with random signs, one does not get the power law growth of Eq. (5), but instead a narrower distribution uniform near zero. Under this assumption, and assuming some

[5] v3: In the borderline case $2n_F + n_D = 2$, the outcome depends on more detailed properties of the distributions.

models solve the μ problem, and combining this with Eq. (6), in the end low scale breaking would be favored.

Note added in v4 (following communications with M. Dine, T. Banks, and S. Shenker). There is a far more serious error in the argument of section 3. Namely, we considered only the distribution in a sector in which all supersymmetry breaking parameters are nonzero. One also expects to find many partially supersymmetric sectors in which only some of the breaking parameters are nonzero, with the rest zero or very small, for reasons explained in [7] section 4 (since the equations $V' = 0$ are quadratic, their solutions lie on several branches, labelled by the rank of a matrix constructed from the breaking parameters).

While a proper treatment probably requires going into details, a simple illustration of the possible effects of this is to take the distribution of breaking parameters suggested in section 2 (uniform with a component at hierarchically small scales) and add a third component which is a delta function at zero. For purposes of this argument, the component at hierarchically small scales could also be taken at zero, leading to $d\mu[D] = \delta(D) + c dD|_0^1$ or $d\mu[F] = \delta(F) + c d(F^2)|_0^1$, with the parameter c expressing the ratio of the number of supersymmetry breaking vacua to the sum of partially supersymmetric vacua and low breaking scale vacua (in this parameter), and the cutoff taken to be 1. Based on the results and heuristic arguments in [7], one might expect $c \sim 1$.

Convolving these distibutions gives a bimodal distribution with peaks both at the low and high scale, with (very roughly) $d\mu[1] \sim (1+c)^{n_D}(1+2c)^{n_F}$. This can compensate the tuning factor M_H^2/M_{susy}^2, but only with many breaking terms, $n \sim |\log_{1+c} M_H^2|$

In words, the simplest reason behind this is that for high scales to dominate, one needs to start with many more high scale vacua than low scale or supersymmetric vacua, to compensate for the advantage of low scale supersymmetry. Since typically $n > 100$ in Calabi-Yau compactification of string theory, even the revised estimate allows this to come out, but clearly a believable argument requires more detail than the simple scaling argument of section 2. It will be quite interesting to carry out a discussion of supersymmetry breaking flux vacua as in [7] for multiparameter models.

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